**Homework 9**



**P18.4.5** Determine *vS* at *t* = 0+ in Figure P18.4.5, given *iSRC* = 20*δ*(*t*) A, and assuming the inductor and capacitor are initially uncharged.

**Solution:** The capacitor presents a short circuit to the current impulse and the inductor an open circuit. The current impulse flows through *C* and the 1 Ω resistor across the inductor, depositing a charge of 20 C across *C* and establishing a flux linkage of 20 WB-T across the inductor. *vC*(0+) = 20/0.5 = 40 V, and *iL*(0+) = -20 A. This current instantaneously results in vL(0+) = -20 V. It follows that *vS* = *vC* – (-*vL*) = 20 V.



**P18.4.9** Determine *vL*, *vC*, *i*, and *vx* at *t* = 0+ in Figure P18.4.9, given *vSRC* = 0.1*u*(*t*) V, and assuming *ρ* = 8 mA/V, and the inductor and capacitor are initially uncharged.

**Solution:** The inductor presents an open circuit to the voltage step and the capacitor a short circuit. The voltage step appears across the inductor, so that *vL*(0+) = 0.1 V and *vC*(0+) = 0. The current in the circuit is zero at *t* = 0+, and *vx*(0+) = 0.

**P18.4.10** Determine *vO*, *vL*, *iL*, and *vx* at *t* = 0+ in Figure P18.4.10, given *iSRC* = 10*u*(*t*) A, and assuming the capacitor is initially uncharged and the inductor has an initial current of 0.1 A.



**Solution:** The voltage across the capacitor and the current through the inductor are not changed at *t* = 0+ by the current step. Hence, *vx*(0+) = 0 and *iL*(0+) = 0.1 A. The dependent source constrains *vO* to be zero at *t* = 0+. This means that *vL*(0+) = -0.1 V, and the current though the capacitor at *t* = 0+ is zero. From KCL, the current through the dependent source at *t* = 0+ is 10 – 0.1 = 9.9 A.



**P19.1.3** The switches in Figure P19.1.3 open at *t* = 0 after having been closed for a long time. Determine the final values of *vC*1 and *vC*2.

**Solution:** *v*1(0-) = 15×12/15 = 12 V, *q*1(0-) = 12 μC, *v*2(0-) = 0, and *q*2(0-) = 0. The circuit at *t* = 0+ is a shown. *Ceq* = 2/3 μF having a voltage of 12 V across it. *qeq* = 8 μC. When the capacitor discharges, the 8 μC circulate clockwise. *q*1*f* = 12 – 8 = 4 μC, and *v*1*f* = 4 V. *q*2*f* = -8 μC, and *v*2*f* = -4 V



**P19.1.6** The capacitors in Figure P19.1.6 were initially charged as shown. When the switch is closed at  the current  is found to be 0.18*e–t* mA where *t* is in ms. Determine: (a)  for *t* ≥ 0+; (b) *R*; and (c) the final voltages across the capacitors.



**Solution:** We first determine the equivalent series capacitor. 60/13 nF in series with 5 nF gives *Ceqs* = nF. The voltage across this capacitor is 60 + 15 = 75 V. , where the minus sign applies because *iφ* is in a direction to discharge *Ceqs*. This gives:  V.

To determine what is inside the box, we note that *τ* = *CR* = 1 ms = . This gives *R* = kΩ. This is the same as , as it should be.

The initial voltage on *Ceqs* is 75 V, and the final voltage is 0. The charge on *Ceqs* has therefore decreased by 2.4×75 ≡ 0.18 μC; this means that charge on the capacitors that are in series between the terminals of the black box has also decreased by this amount of charge. Considering the 60/13 nF capacitor first, a reduction in charge of 0.18 μC causes a reduction in voltage of 39 V. The final voltage is 15 – 39 = -24 V. A reduction in charge of 0.18 μC causes a reduction in voltage of 36 V across the combined 5 nF capacitor. The final voltage on this capacitor is 60 – 36 = 24 V. The final voltages on the capacitors add to 0, as they should.

**P20.1.3** Convolve with .

**Solution:**  = 1 + *e*-2*t* − 2*e*-*t*.

**P20.1.8** Evaluate *t*2*u*(*t*)\*cos*tu*(*t*).

**Solution:** . From a table of integrals (Appendix A2),  = 2*t*cos*t* –



(*t*2 – 2)sin*t*;  = 2*t*sin*t* – (*t*2 – 2)cos*t – 2.* Multiplying the first integral by cost and the second integral by sint and adding, the terms in (*t*2 – 2) cancel out, giving 2*t*cos2*t* + 2*t*sin2*t* – 2sin*t* = 2(*t* – sin*t*).



**P20.2.5** Given *f*(*t*) and *g*(*t*) as in Figure P20.2.5, where *g*(*t*) = sin(*πt*/2), 0 ≤ *t* ≤ 2, and *g*(*t*) = 0 elsewhere. Determine *f*(*t*)\**g*(*t*) for all *t*. Verify the result by direct integration.

**Solution:** *Method 1* – *Graphical Solution*: When *f*(*t*) is folded



around the vertical axis and shifted by *t*, there are

four ranges of *t* to consider:

i) 0 ≤ *t* ≤ 1: *y*(*t*) = = .



ii) 1 ≤ *t* ≤ 2: *y*(*t*) = =

 .



iii) 2 ≤ *t* ≤ 3: *y*(*t*) =  ==



.

iv) *t* ≥ 3: there is no overlap between the

functions *y*(*t*) = 0.

*y*(*t*) will be as shown.

*Method 2* – *Analytical*: The square pulse is expressed as *u*(*t*) – *u*(*t* – 1), and the half-period as . Then *y*(*t*) is: 

= 

.

i) 0 ≤ *t* ≤ 1: Using Equation 20.4.1: 

, as previously.

ii) 1 ≤ *t* ≤ 2: Using Equation 20.4.3, with *b* = 0 and *a* = 1,  . Adding this to the result of (i) gives , as previously.

iii) 2 ≤ *t* ≤ 3: Using Equation 20.4.3, with *b* = 2 and *a* = 0, =  = . Adding this to the result of (ii) gives , as previously.

iv) *t* ≥ 3: Using Equation 20.4.3, with *b* = 2 and *a* = 1,    Adding this to the result of (iii) gives 0, as previously.

*Method 3* – *Differentiating one function and integrating the other*: Differentiating the square pulse gives: *δ*(*t*) – *δ*(*t* – 1). The integral of the half period is as shown,



and can be expressed as:  . Then *y*(*t*) is: .

i) 0 ≤ *t* ≤ 1: Using Equation 20.4.6: , as previously, bearing in mind that .

ii) 1 ≤ *t* ≤ 2: Using Equation 20.4.9, with *b* = 0 and *a* = 1: . Adding this to the result of (i) gives , as previously.

iii) 2 ≤ *t* ≤ 3: Using Equation 20.4.9, with *b* = 2 and *a* = 0: . Adding this to the result of (i) gives , as previously.

iv) *t* ≥ 3: Using Equation 20.4.9, with *b* = 2 and *a* = 1:  . Adding this to the result of (i) gives 0, as previously.